

Questions 161 - 180

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

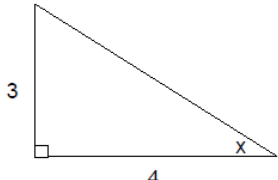
$$= 1 - 2 \sin^2 A$$

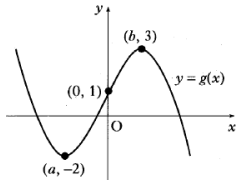
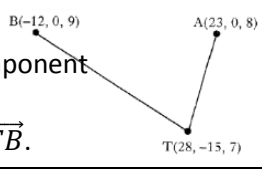
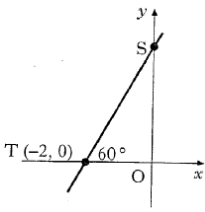
Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

161 Show that $(x + 2)$ is a factor of $f(x) = x^3 - 2x^2 - 4x + 8$ and hence fully factorise $f(x)$.	
162 Calculate the discriminant of the quadratic equation $2x^2 + 4x + 5 = 0$	
163 Solve the equation $\log_4(5 - x) - \log_4(3 - x) = 2$, $x < 3$.	
164 Find all the values of x in the interval $0 \leq x \leq 2\pi$ for which $\tan^2(x) = 3$.	
165 Show that the exact value of $\cos 2x$ is $\frac{7}{25}$.	
166 If $y = 3\cos^4 x$, find $\frac{dy}{dx}$.	
167 A curve has equation $y = x^3 - 3x^2 - 9x + 12$. Find the coordinates of the stationary points of this curve and determine their nature.	
168 Find $\int \frac{4x^3 - 1}{x^2} dx, x \neq 0$.	
169 Find the value of $\int_0^2 \sin(4x + 1) dx$.	
170 A curve has equation $y = 7\sin x - 24\cos x$. Express $7\sin x - 24\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq \frac{\pi}{2}$.	

<p>171 $f(x) = 3x - 1$ and $g(x) = \frac{1}{x+1}$ Find $f(g(x))$ and $g(f(x))$. State a suitable domain for $g(f(x))$.</p>	
<p>172 The diagram shows the graph $y = g(x)$.</p> <p>a. Sketch $y = -g(x)$ b. Sketch $y = 3 - g(x)$</p>	
<p>173 If $\mathbf{f} = 3\mathbf{i} + 2\mathbf{k}$ and $\mathbf{g} = 2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, Find $\mathbf{f} + \mathbf{g}$.</p>	
<p>174 Express the vectors \overrightarrow{TA} and \overrightarrow{TB} in component form. Calculate the angle between \overrightarrow{TA} and \overrightarrow{TB}.</p>	
<p>175 Prove the identity: $(\cos P^\circ + \sin P^\circ)^2 = 2\sin P^\circ \cos P^\circ + 1$</p>	
<p>176 Find the equation of the line ST, where T is the point $(-2, 0)$ and angle STO is 60°.</p>	
<p>177 Triangle ABC has vertices $A(-1, 12)$, $B(-2, -5)$ and $C(7, -2)$. Find the equation of the altitude AE.</p>	
<p>178 Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle.</p>	
<p>179 A sequence is defined by the recurrence relation $u_{n+1} = 0.2u_n + 5$ with $u_8 = 20$. Calculate u_{10}.</p>	
<p>180 Calculate the area enclosed between the curve $y = x^2 - 6x$ and the x-axis.</p>	