

Questions 41 - 60

Answers Included!

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

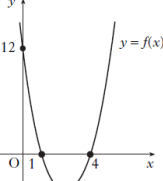
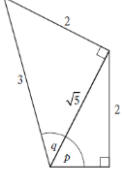
$$= 1 - 2 \sin^2 A$$

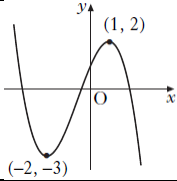
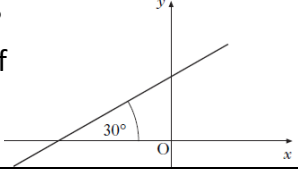
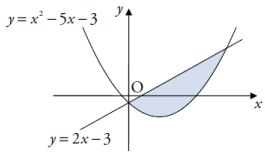
Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

<p>41 The graph has an equation of the form $y = k(x - a)(x - b)$. What is the equation of the graph?</p>		
<p>42 For what values of x is $6 + x - x^2 < 0$?</p>		
<p>43 Express $\log_a 25 + \log_a 4 - \log_a 20$ as the logarithm of a single number.</p>		
<p>44 Solve $\cos 2x - 3\cos x + 2 = 0$ for $0 \leq x < 360$.</p>		
<p>45 The diagram shows two right-angled triangles with sides and angles given. What is the value of $\sin(p + q)$?</p>		
<p>46 What is the derivative of $(x^3 + 4)^2$?</p>		
<p>47 The point $P(5, 12)$ lies on the curve with equation $y = x^2 - 4x + 7$. Find the equation of the tangent to the curve.</p>		
<p>48 Find $\int 4 \sin(2x + 3) dx$.</p>		
<p>49 Find $\int_{-2}^2 (x + 1)^2 dx$.</p>		
<p>50 Write $2\sin x^\circ + 3\cos x^\circ$ in the form $k\sin(x - a)$, for $k > 0$ and $0 \leq a \leq 360$.</p>		

<p>51 Functions f and g are defined on a suitable domain by $f(x) = \cos x$ and $g(x) = x + \frac{\pi}{6}$. What is the value of $f\left(g\left(\frac{\pi}{6}\right)\right)$?</p>	
<p>52 The diagram shows the graph of $y = f(x)$. Sketch $y = f(x + 2) - 1$</p>	
<p>53 Given that $\mathbf{u} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$, find $3\mathbf{u} - 2\mathbf{v}$ in component form.</p>	
<p>54 The vectors $x\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$ and $-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ are perpendicular. What is the value of x?</p>	
<p>55 Prove that $2\cos^2 A + 3\sin^2 A - 2 = \sin^2 A$.</p>	
<p>56 A line makes an angle of 30° with the positive direction of the x-axis as shown. What is the gradient of the line?</p>	
<p>57 Find the equation of the perpendicular bisector of the line joining $P(3, -3)$ to $Q(-1, 9)$.</p>	
<p>58 Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$</p>	
<p>59 A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 3$ and $u_0 = 1$. What is the value of u_3?</p>	
<p>60 Calculate the shaded area enclosed by the line $y = 2x - 3$ and the curve $y = x^2 - 5x - 3$.</p>	

Answers Only

Ques 1 - 20	Ques 21 - 40
<ol style="list-style-type: none"> 1. $f(x) = (x - 1)(2x + 5)(x - 1)$ 2. $(x + 4)^2 - 13$ Min T.P at $(-4, -13)$ 3. 2 4. $x = \frac{2\pi}{3}$ for $\frac{\pi}{2} \leq x \leq \pi$ 5. $\sin(x + a) = \frac{4}{5}\sin x + \frac{3}{5}\cos x$ 6. $\frac{dy}{dx} = 12x^2 + 10x - 3$ 7. Max T.P at $(-1, 17)$ and Min T.P. at $(3, -15)$ 8. $\frac{-2x^{-3}}{3} + \frac{1}{5}\sin 5x + C$ 9. $y = 4x^2 - 3x - 3$ 10. $k = 2$ and $a = 30^\circ$ 11. $x \leq 3$ 12. $a = 2$ $b = 3$ 13. 1:2 14. 26 15. $\sin 60 = \frac{\sqrt{3}}{2}$ $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$ 16. $m = -1$ 17. $y = -3x + 10$ 18. $(x + 7)^2 + (y - 6)^2 = 36$ 19. $u_{12} = 8.7$ 20. Area = 32 square units 	<ol style="list-style-type: none"> 21. $f(x) = (x - 1)(x + 2)(x - 1)$ 22. $q = 5$ 23. $x = 2$ 24. $x = \frac{\pi}{6}, \frac{11\pi}{6}$ 25. $\cos 2x = \frac{-3}{5}$ 26. $f'(x) = 3x(4 - 3x^2)^{-\frac{3}{2}}$ 27. Max T.P. at $(-1, 4)$ and Min T.P. at $(1, 0)$ 28. $\frac{8}{3}x^{\frac{3}{2}} - \frac{1}{2}x^{-2} + C$ 29. $y = \frac{-1}{3}\cos 3x + \frac{7}{6}$ 30. $\sqrt{2}\sin(x - \frac{\pi}{4})$ 31. $f(g(x)) = 3x^2 - 5$ $g(f(x)) = 9x^2 + 6x - 1$ 32. $a = 3$ $b = 3$ 33. $k = 4$ 34. $\overrightarrow{DE} = 3\overrightarrow{EF}$ so \overrightarrow{DE} and \overrightarrow{EF} are parallel. E is a common point so D,E,F are collinear. 35. Proof. 36. $3y + 5x = -13$ 37. $m_{PS} = \frac{7}{4}$ 38. $4y + 5x = 71$ 39. $L = -400$ 40. Area = $12\frac{3}{20}$ square units.
Ques 41 - 60	Ques 61 - 80
<ol style="list-style-type: none"> 41. $Y = 3(x - 1)(x - 4)$ 42. $x > 3$ $x < -2$ 43. $\log_a 5$ 44. $x = 0^\circ, 60^\circ, 300^\circ, 360^\circ$ 45. $\sin(p + q) = \frac{2+2\sqrt{5}}{3\sqrt{5}}$ 46. $\frac{dy}{dx} = 6x^5 + 24x^2$ 47. $y = 6x - 18$ 48. $-2\cos(2x + 3) + C$ 49. $9\frac{1}{3}$ 50. $\sqrt{13}\sin(x - 303.7)$ 51. 0.5 52. Correct shape, Min T.P at $(-4, -4)$ Max T.P. at $(-1, 1)$ 53. $\begin{pmatrix} 8 \\ -4 \\ -5 \end{pmatrix}$ 54. $x = 1$ 55. Proof 56. $m = \frac{1}{\sqrt{3}}$ 57. $3y - x - 8 = 0$ 58. Centre $(-4, -2)$ radius = $\sqrt{58}$ 59. $u_3 = 29$ 	<ol style="list-style-type: none"> 61. $(x - 1)(x + 4)(x + 5)$ 62. $k = \frac{9}{8}$ 63. -4 64. $x = 60^\circ, 132^\circ, 228^\circ, 300^\circ$ 65. $\cos 2a = \frac{7}{25}$ 66. 14π 67. $y = -x + 1$ 68. $\frac{x^{-3}}{-9} + C$ 69. 1 70. $5\cos(x + 306.9^\circ)$ 71. $f(g(x)) = x^2 + 8x + 19$ $g(f(x)) = x^2 + 7$ 72. $q = 13$ 73. 3:2 74. $\frac{9}{2}$ 75. $120^\circ = \frac{3\pi}{4}$ and $\frac{2\pi}{3} = 120^\circ$ 76. 5 units 77. $y = 4x + 4$ 78. $J(-1, -2)$ $K(1, 2)$ 79. $l = 100/3$ 80. Area = $\frac{27}{4}$ square units

60. Area = $57\frac{1}{6}$ square units	
<p style="text-align: center;">Ques 81 - 100</p> <p>81. $K = -2$ and $t = -5$ 82. $x > 0$ $x < -4$ 83. $x = \frac{3}{8}$ 84. $x = 30^\circ, 90^\circ, 150^\circ$ 85. $\sin(a + b) = \frac{63}{65}$ 86. $f'(x) = -\frac{1}{5}x^{-\frac{6}{5}}$ 87. $y = 4x - 2$ 88. $\frac{(2x-1)^{\frac{3}{2}}}{3} + C$ 89. $\frac{2}{3}$ 90. $\sqrt{34}\cos(x - 59.0)^\circ$ 91. $f(g(x)) = 9x^2 - 24x + 17$ $g(f(x)) = 3x^2 - 1$ 92. $a = 4$ $b = 2$ $c = 1$ 93. 3:2 94. $\theta = 72^\circ$ 95. -1 96. $y = \frac{1}{4}x + \frac{13}{4}$ 97. $y = \frac{3}{2}x - 3$ 98. Centre $(-1, -2)$ Radius $\sqrt{32}$ 99. $L = \frac{28}{3}$ 100. Area = 9 square units</p>	<p style="text-align: center;">Ques 101 - 120</p> <p>101. 4 102. $q = -9$ 103. $x = 2y^3$ 104. $x = 20^\circ, 100^\circ, 140^\circ, 220^\circ, 260^\circ, 340^\circ$ 105. $\sin 2x = \frac{4}{5}$ 106. 1 107. Max T.P. when $x = \frac{1}{3}$ 108. $x^3 + x^2 + c$ 109. $f(x) = 2x + \frac{1}{x} + 5$ 110. $2.5\cos(x + 306.9)^\circ$ 111. $x \neq 3$ and $x \neq 4$ 112. Correct shape drawn and labelled with $(0,3), (3,1), (5,3)$ 113. $Q(3, 1, -2)$ 114. $t = -3$ 115. Proof. 116. $m = 1$ 117. $y = -2x + 2$ 118. $y = -\frac{3}{2}x + 6$ 119. $u_4 = 3$ 120. $9/8$</p>
<p style="text-align: center;">Ques 121 - 140</p> <p>121. $y = -x(x + 1)(x - 2)$ 122. $x < -5$ and $x > 3$ 123. 1 124. $x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$ 125. Proof. 126. 12 127. $y = 2x - 12$ 128. $-\frac{(1-6x)^{\frac{1}{2}}}{3} + C$ 129. $y = 2x^3 - 2x^2 + 3x + 2$ 130. $10\cos(x + 36.9)^\circ$ 131. $p(x) = 3 - \frac{3}{x}$ and $p(q(x)) = x$ 132. $y = -2f(x)$ passing through $(-6, 0), (1, 14), (3, 0)$ and $y = f(x - 3)$ passing through $(-3, 0), (4, -7), (6, 0)$ 133. $\vec{QR} = 2\vec{PQ}$ and Q is a common point so P, Q, R are collinear. 134. 5 135. Proof. 136. $y = -\frac{1}{2}x + \frac{13}{2}$ 137. $y = 6x + 9$ 138. $y = \frac{1}{4}x - \frac{7}{2}$ 139. $L = -50$ 140. Area = $\frac{20}{3}$ square units</p>	<p style="text-align: center;">Ques 141 - 160</p> <p>141. $(x - 4)(x - 2)(x + 1)$ 142. $x < -3$ and $x > 2$ 143. $k = 0.46$ 144. $x = 45^\circ, 105^\circ, 225^\circ, 285^\circ$ 145. Proof. 146. $\frac{dy}{dx} = -6x^{-3} + 3x^{\frac{1}{2}}$ 147. $y = 2x - 11$ 148. $\frac{x^3}{3} + 4x^{-1} + C$ 149. $y = 2x^2 - 2x^3 + 5$ 150. $x = 53.1^\circ, 413.1^\circ$ 151. $f(g(x)) = 8x^2 + 80x + 195$ $g(f(x)) = 8x^2$ 152. $y = f(x - 4)$ passing through $(0,5), (5, a)$ $y = 2 + f(x - 4)$ passing through $(0,7), (5, a + 2)$ 153. AB:BC = 1:2 154. Unit vector = $\begin{pmatrix} \frac{4}{5} \\ 0 \\ -\frac{3}{5} \end{pmatrix}$ 155. Proof. 156. $m = 3$ 157. $y = x$ 158. $(x + 2)^2 + (y - 3)^2 = 18$ 159. $L = 60$ 160. Area = $\frac{343}{6}$ square units</p>

Ques 161 - 180	Ques 181 - 200
161. $(x + 2)(x - 2)(x - 2)$ 162. $b^2 - 4ac = -24$ 163. $x = \frac{43}{15}$ 164. $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ 165. Proof. 166. $\frac{dy}{dx} = -12\sin x \cos^3 x$ 167. Max T.P at (-1, 17). Min T.P at (3, -15). 168. $2x^2 + \frac{1}{x} + C$ 169. 0.363 radians 170. $25\sin(x - 1.287)$ (in radians) 171. $f(g(x)) = \frac{-x+2}{x+1}$ $g(f(x)) = \frac{1}{3x}$ 172. $y = -g(x)$ passes through $(a, -2), (0, -1), (b, -3)$ $y = 3 - g(x)$ passes through $(a, 5), (0, 2), (b, 0)$ 173. $ f + g = \sqrt{66}$ 174. $\theta = 50.9$ 175. Proof. 176. $y = \sqrt{3}x + 2\sqrt{3}$ 177. $y = -3x + 9$ 178. Point of contact is (1, 4) 179. $u_9 = 9$ $u_{10} = 6.8$ 180. Area = 36 square units	181. $3(3x + 1)(x + 2)(x - 1)$ 182. $-1 < x < \frac{1}{3}$ 183. $x = 71$ 184. $x = 45^\circ, 105^\circ, 165^\circ, 225^\circ, 285^\circ, 345^\circ$ 185. $\sin 2a = \frac{15}{17}$ 186. $\frac{dy}{dx} = 2x \cos(x^2 - 3)$ 187. Min T.P at (0, 0) Max T.P at (2, 4) 188. $\frac{(2x+9)^6}{12} + C$ 189. $\frac{13}{3}$ 190. $2\sin(x + \frac{\pi}{3})$ 191. $x \neq -8, x \neq 2$ 192. $y = 2f(x) + 1$ passes through (0, 1), (2, 7), (5, 1) 193. $\vec{PQ} = \begin{pmatrix} 8 \\ -4 \\ 1 \end{pmatrix}$ $ \vec{PQ} = 9$ 194. $k = \frac{6}{7}$ 195. Proof. 196. $y = 3x + 7$ 197. $3y = 4x - 11$ 198. $P(1, -2)$ $Q(3, 4)$ 199. $u_1 = 16$ $u_2 = 20$ $u_3 = 21$ 200. Area = $\frac{9}{2}$ square units

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