



Higher  
Mathematics  
Practice Exam  
2023

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**Essential Skills (1 hour 15 minutes)**

**Mathematics  
Paper 1 (Non-calculator)**

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Total marks – 55

Attempt ALL questions.

**You may NOT use a calculator.**

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Use blue or black ink.

## FORMULAE LIST

### Circle

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Scalar product

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ , where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

### Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

### Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

### Table of standard integrals

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
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**Attempt ALL questions**  
**Total marks – 55**

1. A function  $g$  is defined by  $g(x) = \frac{3x-1}{4}$ , where  $x \in \mathbb{R}$ .
- Find the inverse function,  $g^{-1}(x)$ . **3**
- 
2. Given  $h(x) = 3 \cos 2x$ , find the value of  $h'(\frac{3\pi}{4})$ . **3**
- 
3. Find the equation of the line perpendicular bisector to the line joining  $A(2, -4)$  to  $B(-2, 8)$ . **4**
- 
4. Functions  $f$  and  $g$  are defined by
- $f(x) = \frac{1}{\sqrt{x}}$ , where  $x > 0$ .
  - $g(x) = x^2 - 2x - 8$ , where  $x \in \mathbb{R}$ .
- (a) Determine an expression for  $h(x)$  where  $h(x) = f(g(x))$ . **2**
- (b) State the domain of  $h(x)$ . **3**
- 
5. (a) Show that  $(x - 1)$  is a factor of  $f(x) = 2x^3 - 11x^2 + 4x + 5$ . **2**
- (b) Hence, factorise  $f(x)$  fully. **2**

6. A ball is thrown upwards. The height  $h$  metres of the ball  $t$  seconds after it is thrown, is given by the formula  $h = 18t - 3t^2$ .

(a) Find the rate of change of the ball when it is thrown. 3

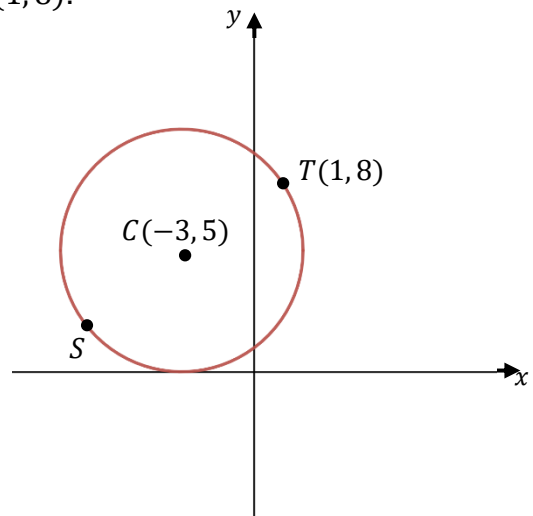
(b) Find the speed of the ball after 3 seconds.  
Explain your answer in terms of movement of the ball. 2

7. Solve  $\log_3(x + 2) + \log_3(2x + 1) = 2$ , where  $x > -\frac{1}{2}$ . 4

8. The circle has centre  $C(-3, 5)$  and passes through  $T(1, 8)$ .

(a) Find the equation of the circle. 2

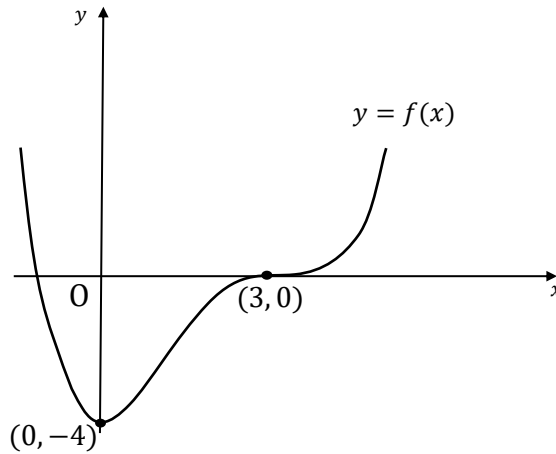
(b)  $ST$  is the diameter of the circle. Find the equation of the tangent to this circle at  $S$ . 4



9. (a) Find the  $x$ -coordinate of the stationary point on the curve with equation  $f(x) = 2\sqrt{x^3} - 6x$ . 4

(b) Hence, determine the greatest and least values of  $y$  in the interval  $1 \leq x \leq 9$ . 3

10. The diagram below shows the graph of  $y = f(x)$ .



On separate diagrams, sketch the graphs of:

(a)  $y = -f(x + 3)$ .

2

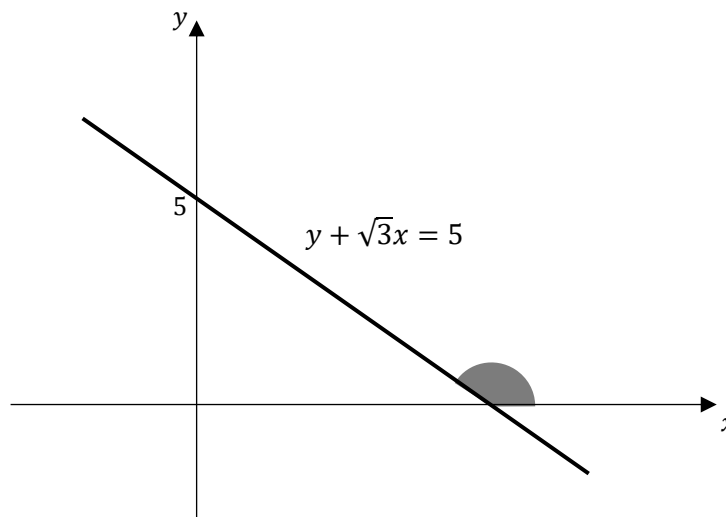
(b)  $y = f'(x)$ .

3

11. A line has equation  $y + \sqrt{3}x = 5$ .

Determine the angle the line makes with the positive direction of the  $x$ -axis.

3



12. (a) Express  $3x^2 - 6x + 4$  in the form  $a(x + b)^2 + c$ .

3

(b) Hence, show that  $y = x^3 - 3x^2 + 4x - 5$  is strictly increasing for all  $x \in \mathbb{R}$ .

3

[ END OF QUESTION PAPER ]

# Paper 1 - Answers

1.  $g^{-1}(x) = \frac{4x+1}{3}$
2. 6
3.  $y = x/3 + 2$
4. (a)  $\frac{1}{\sqrt{x^2-2x-8}}$  (b)  $x < -2, x > 4$
5. (a) Remainder zero (b)  $(x-1)(2x+1)(x-5)$
6. (a) 18m/s (b) zero, stationary
7.  $x = 1$
8. (a)  $(x+3)^2 + (y-5)^2 = 25$  (b)  $3y + 4x + 22 = 0$
9. (a)  $x = 4$  (b) Greatest 0, when  $x = 9$ ; Least  $-8$ , when  $x = 4$
10. (a) Flipped (b) Derived Graph
11.  $\theta = 120^\circ$
12. (a)  $3(x-1)^2 + 1$  (b)  $\frac{dy}{dx} > 0 \forall x$ , so function strictly increasing



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**Essential Skills (1 hour 30 minutes)**

**Mathematics  
Paper 2**

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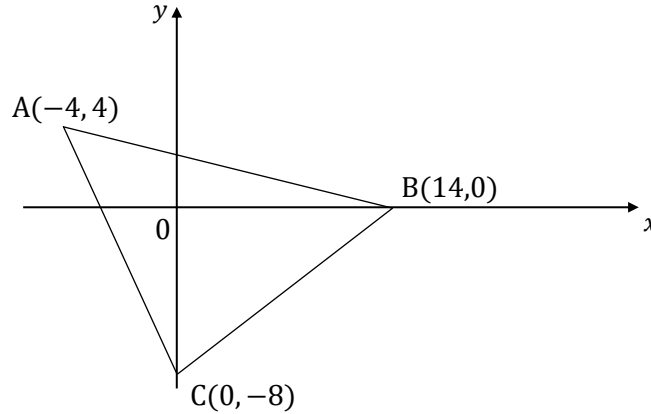
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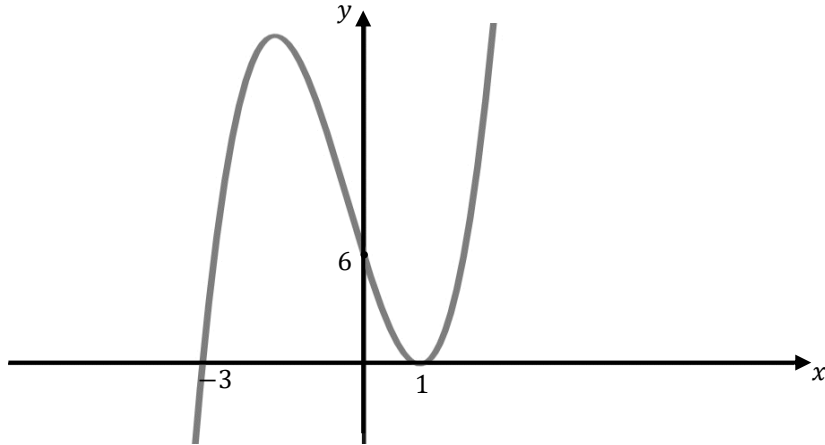
Attempt ALL questions  
Total marks – 65

1. The vertices of triangle PQR are  $A(-4, 4)$ ,  $B(14, 0)$  and  $C(0, -8)$  as shown in the diagram.



- (a) Find the equation of the median from C. 3
- (b) Find the equation of the altitude from B. 3
- (c) Find the coordinates of the point of intersection of the median from C and altitude from B. 2
2. (a) Express  $3 \cos x^\circ + \sin x^\circ$  in the form  $k \cos(x - \alpha)^\circ$ , where  $k > 0$  and  $0 \leq \alpha < 360$ . 4
- (b) A function,  $f$ , is defined by  $f(x) = 3 \cos x^\circ + \sin x^\circ$ ,  $0 \leq x < 360$ .
- (i) State the minimum value of  $f(x)$ . 1
- (ii) Determine the value of  $x$  where this minimum occurs. 1
3. The equation  $x^2 + (k - 1)x + 4 - k = 0$  has equal roots.  
Determine the possible values of  $k$ . 3
4. For a function,  $f$ , defined on the set of real numbers,  $\mathbb{R}$ , it is known that
- the gradient of the tangent to the curve,  $f$ , with respect to  $x$  is given by  $12x^2 - 2x$ .
  - the curve  $y = f(x)$  passes through the point  $(1, 4)$ .
- Express  $f(x)$  in terms of  $x$ . 5

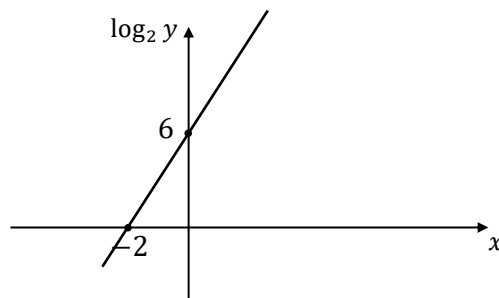
5. The diagram shows the graph with the equation  $f(x) = k(x - a)(x - b)^2$ .



Find the values  $a, b$  and  $k$ .

3

6. Two variables,  $x$  and  $y$ , are connected by the equation  $y = ab^x$ .  
The graph of  $\log_2 y$  against  $x$  is a straight line as shown.



Find the values of  $a$  and  $b$ .

5

7. Solve the equation

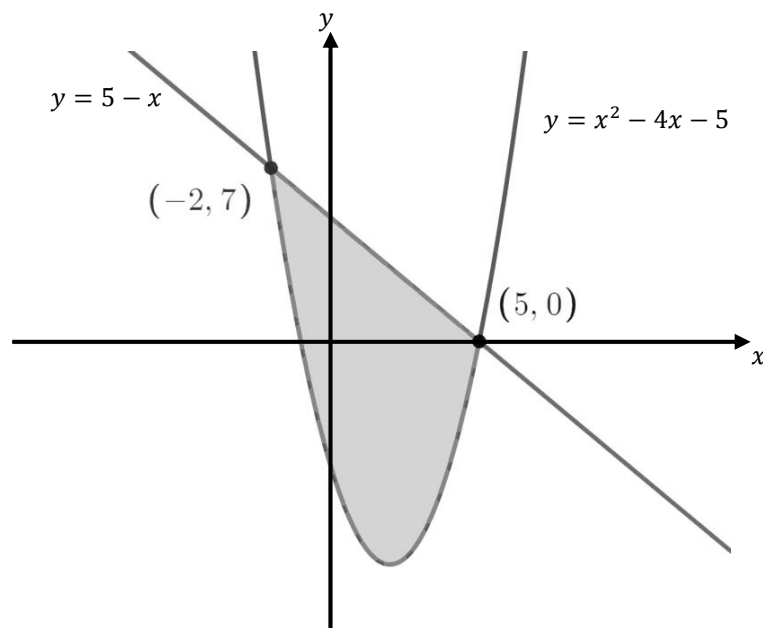
$$3\sin 2x - 2\cos x = 0 \quad \text{for } 0 \leq x \leq 2\pi.$$

5

8. Determine the coordinates of the points of intersection between the line with equation  $x = 3y - 30$  and the circle with equation  $x^2 + y^2 + 18x - 4y - 140 = 0$ .

5

9. The graph of the curve  $y = x^2 - 4x - 5$  and line  $y = 5 - x$  are shown below.



They intersect at  $(-2, 7)$  and  $(5, 0)$ .

Calculate the area enclosed by the line and curve.

5

10. A governing body provides a shipbuilder with a contract to build a new passenger and vehicle ferry.

The costs and overall value of the new liner will be £44 million.

It is estimated that the depreciation rate of the liner will be modelled by the formula,

$$V_t = V_0 e^{-0.042t}$$

where:

- $V_0$  is the initial value
- $V_t$  is the value at time,  $t$
- $t$  is the time, in years, since launch

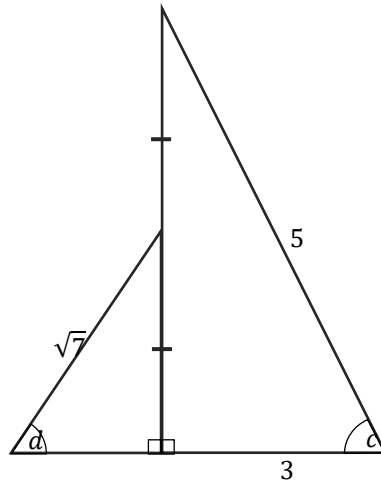
Due to aging a ferry should be removed from service when the value drops to £11 million.

The governing body claim this will give the ferry a life of 35 years.

Is this claim accurate?

5

11. Two right angled triangles are shown below.



(a) Determine the value of

(i)  $\sin c$

1

(ii)  $\cos d$

2

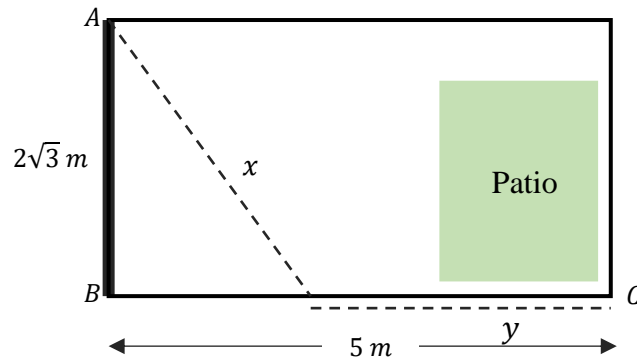
(b) Find the exact value of  $\sin (c + d)$ .

3

12. A rectangular garden of a property measures 5 metres by  $2\sqrt{3}$  metres.

New copper piping to supply gas to the property has to be laid between the points A and C.

The owner doesn't want their front wall, AB, or patio dug to provide this service so the following plan is drawn.



The cost of laying piping through the garden is twice the cost, per metre, of going around the perimeter.

(a) Show that the total cost of laying this piping can be modelled by

$$C(x) = 2x + 5 - \sqrt{x^2 - 12}, \quad (x > \sqrt{12})$$

3

(b) Find the value of  $x$  which would minimise the cost of laying this piping.

6

[ END OF QUESTION PAPER ]

**Answers**

1. (a)  $y = 2x - 8$  (b)  $3y = x - 14$  (c)  $(2, -4)$
2. (a)  $\sqrt{10}\cos(x - 18.4^\circ)$  (b)  $-\sqrt{10}, x = 198.4^\circ$
3.  $k = -5, k = 3$
4.  $y = 4x^3 - x^2 + 1$
5.  $a = -3, b = 1, k = 2$
6.  $a = 64, b = 8$
7.  $x = 0.340, \frac{\pi}{2}, 2.802, \frac{3\pi}{2}$
8.  $(-24, 2)$  and  $(3, 11)$
9.  $\frac{343}{6} \text{ units}^2$
10. 33 years, claim is not accurate.
11. (a)  $\text{sinc} = \frac{4}{5}, \text{cosd} = \frac{\sqrt{3}}{\sqrt{7}}$  (b)  $\frac{4\sqrt{3}+6}{5\sqrt{7}}$
12. (a) Proof (b)  $x = 4 \text{ metres}$