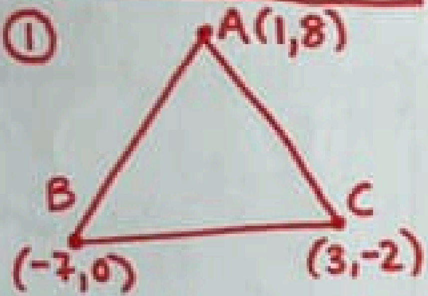
HIGHER MATHS 2024 LAST MINUTE NON CALCULATOR

① 

a) Find the equation of the median from A.

b) Find the equation of the altitude from C.

c) Find the point of intersection of the two lines in (a) and (b).

d) Show that the midpoint of AB lies on the altitude from C and hence make a statement about the triangle ABC.

② Simplify

$2\log_6 3 + \log_6 4$

③ Express

$4x^2 - 24x + 35$ in the form $a(x+b)^2 + c$

④ Evaluate

$\int 5\cos(2x - \frac{\pi}{2})\,dx$

⑤ Given that $(x-1)$ is a factor of $2x^4 - x^3 - 9x^2 + 4x + 4$

Solve

$2x^4 + 4x + 4 = x^3 + 9x^2$

⑥ Calculate the rate of change of $f(x) = \frac{2x + 6x^2}{\sqrt{x}}$

when x = 4 [redacted]

⑦ Circle C_1 has equation

$(x-3)^2 + (y+4)^2 = 49$

Circle C_2 has equation

$x^2 + 4x + y^2 - 16y + 59 = 0$

Find the distance between the centres of C_1 and C_2

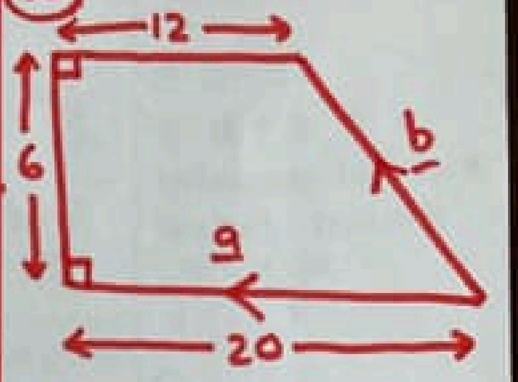
⑧ Determine the exact value of

a) $\sin A$

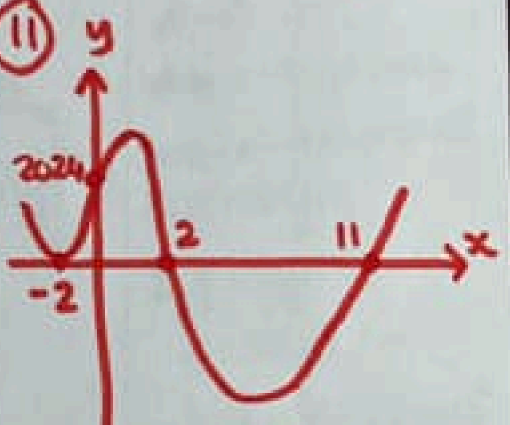
b) $\cos B$

c) $\cos(A+B)$.

⑨ Find $f^{-1}(x)$ given $f(x) = 3x^5 + 1$.

⑩ 

Find $\underline{a}.(\underline{a}+\underline{b})$

⑪ 

The diagram shows the graph of

$y = a(x+b)(x+c)(x+d)^2$

Find the values of a, b, c and d.

⑫ The recurrence relations

$u_{n+1} = \frac{4}{5}u_n - 6$

and

$v_{n+1} = \frac{2}{3}v_n + t$

have the same limit as $n \to \infty$.

Find the value of t.

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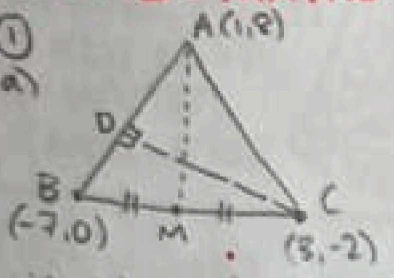
①

a) 

$\text{midpoint}_{BC} = \left(\frac{3+-7}{2}, \frac{0+-2}{2}\right)$

$= (-2,-1)$

$m_{AM} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-(-1)}{1-(-2)}$

$= \frac{9}{3}$

$= 3$

$y - b = m(x-a)$

$y - 8 = 3(x-1)$

$y - 8 = 3x - 3$

$\underline{\underline{y = 3x + 5}}$

b) $m_{AB} = \frac{8-0}{1-(-7)} = \frac{8}{8} = 1$

$m_{CD} = -1$ since $m_1 \times m_2 = -1$ for perpendicular lines!

$y - b = m(x-a)$

$y + 2 = -1(x-3)$

$y + 2 = -x + 3$

$\underline{\underline{y = -x + 1}}$

c) Intersect when y = y

$3x + 5 = -x + 1$

$4x = -4$

$x = -1$

when $x = -1$, $y = 3(-1) + 5 = 2$.

intersection @ $\underline{\underline{(-1,2)}}$

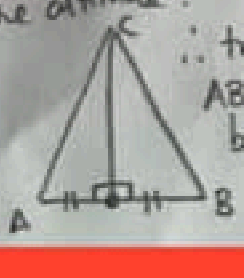
d) $\text{midpoint}_{AB} = \left(\frac{1+(-7)}{2}, \frac{0+8}{2}\right)$

$= (-3,4)$

sub $x = -3$ into $y = -x + 1$

$y = -(-3) + 1 = 4$.

∴ this midpoint lies on the altitude.



∴ triangle ABC must be isosceles!

②

$= \log_6 3^2 + \log_6 4$

$= \log_6 9 + \log_6 4$

$= \log_6 36$

$= \log_6 6^2$

$= 2\log_6 6$

$= \underline{\underline{2}}$

③

$4(x^2 - 6x) + 35$

$4(x-3)^2 + 35 - 36$

$4(x-3)^2 - 1$

$\underline{\underline{a = 4, b = -3, c = -1}}$

④

$\underline{\underline{\frac{5}{2}\sin(2x - \frac{\pi}{2}) + C}}$

⑤

$$\begin{array}{r|rrrrr} 1 & 2 & -1 & -9 & 4 & 4 \\ & \downarrow & 2 & 1 & -8 & -4 \\ \hline & 2 & 1 & -8 & -4 & \underline{0} \end{array}$$

zero remainder so $(x-1)$ is a factor and $x = 1$ is a root.

$$\begin{array}{r|rrrr} 2 & 2 & 1 & -8 & -4 \\ & \downarrow & 4 & 10 & 4 \\ \hline & 2 & 5 & 2 & \underline{0} \end{array}$$

zero remainder so $(x-2)$ is a factor and $x = 2$ is a root

$$\begin{array}{r|rrr} -2 & 2 & 5 & 2 \\ & \downarrow & -4 & -2 \\ \hline & 2 & 1 & \underline{0} \end{array}$$

zero remainder so $(x+2)$ is a factor and $x = -2$ is a root.

Final factor is $(2x+1)$ so final root is $x = -1/2$

solution to the polynomial equation is $\underline{\underline{x = 1, 2, -2, -1/2}}$

⑥

$f(x) = \frac{2x + 6x^2}{x^{1/2}}$

$f(x) = 2x^{\frac{1}{2}} + 6x^{3/2}$

$f'(x) = x^{-1/2} + 9x^{1/2}$

$f'(x) = \frac{1}{\sqrt{x}} + 9\sqrt{x}$

rate of change

$f'(4) = \frac{1}{\sqrt{4}} + 9\sqrt{4} = \underline{\underline{18\frac{1}{2}}}$

⑦

C_1 centre $= (3,-4)$

C_2 centre $= (-2,8)$

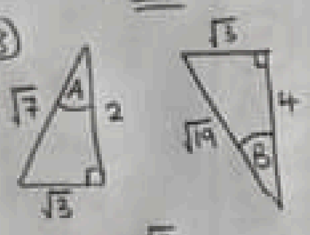
distance from centres $= \sqrt{(8-(-4))^2 + (-2-3)^2}$

$= \sqrt{12^2 + (-5)^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

$= \underline{\underline{13}}$

⑧ 

a) $\sin A = \frac{\sqrt{3}}{\sqrt{7}}$

b) $\cos B = \frac{4}{\sqrt{19}}$

c) $\cos(A+B)$

$= \cos A\cos B - \sin A\sin B$

$= \frac{2}{\sqrt{7}}\frac{4}{\sqrt{19}} - \frac{\sqrt{3}}{\sqrt{7}}\frac{\sqrt{3}}{\sqrt{19}}$

$= \frac{8}{\sqrt{133}} - \frac{3}{\sqrt{133}}$

$= \underline{\underline{\frac{5}{\sqrt{133}}}}$

⑨

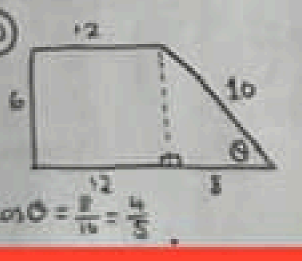
$y = 3x^5 + 1$

$y - 1 = 3x^5$

$\frac{y-1}{3} = x^5$

$\sqrt[5]{\frac{y-1}{3}} = x$

$\underline{\underline{f^{-1}(x) = \sqrt[5]{\frac{x-1}{3}}}}$

⑩ 

$\cos\theta = \frac{8}{10} = \frac{4}{5}$

so

$\underline{a}.(\underline{a}+\underline{b})$

$\underline{a}.\underline{a} + \underline{a}.\underline{b}$

$|a||a|\cos 0 + |a||b|\cos\theta$

$20 \times 20 \times 1 + 20 \times 10 \times \frac{4}{5}$

$400 + 160$

$= \underline{\underline{560}}$

⑪

$b = -11$, $c = -2$ — or vice versa!

$d = 2$

when $x = 0$, $y = 2024$.

$2024 = a(0-11)(0-2)(0+2)^2$

$2024 = 88a$ $\quad \div 8$

$253 = 11a$ $\quad \div 11$

$a = 23$

$y = 23(x-11)(x-2)(x+2)^2$

$\underline{\underline{a = 23, b, c = -11, -2, d = 2}}$

⑫ Limits exist as 4/5 and 2/3 are both between -1 and 1.

$\left.\begin{array}{l} L = \frac{4}{5}L - 6 \\ \frac{1}{5}L = -6 \\ L = -30. \end{array}\right\} u_n$

$\left.\begin{array}{l} L = \frac{2}{3}L + t \\ \frac{1}{3}L = t. \\ \frac{1}{3}(-30) = t \end{array}\right\} v_n$

$\underline{\underline{t = -10}}$

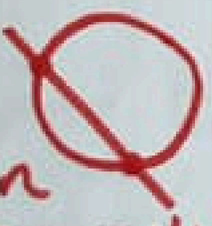
HIGHER MATHS 2024 LAST MINUTE CALCULATOR

① Express $\sqrt{3}\sin x° + \cos x°$ in the form $k\sin(x-a)°$ (where $k>0$, $0<a<360$).

b) Hence, or otherwise, find the maximum value of $y=\sqrt{3}\sin x° + \cos x°$

② Calculate, to 3s.f., the size of the angle between $\underline{a}=\begin{pmatrix}3\\-1\\4\end{pmatrix}$ and $\underline{b}=\begin{pmatrix}-1\\-5\\9\end{pmatrix}$

③ Find the points of intersection of the line $y=x+4$ and the circle $x^2+y^2+8x+2y-96=0$



④ Solve, for $0<x<2\pi$

$5\cos\left(2x+\frac{\pi}{4}\right)=3$

⑤ Find the range of values of p for which $x^2+(p+1)x+9=0$ has real roots.

⑥ The number of tigers world-wide is modelled by the equation

$T_t=4000e^{-kt}$

where T_t is the number of tigers t years after the year 2000.

a) How many tigers were there in the year 2000?

b) By 2010, there were only 3200 tigers worldwide. How many will we expect in 2030?

⑦ $f(x)=3x^2+6x+2$

a) Show that $f(x+1)=3x^2+12x+11$

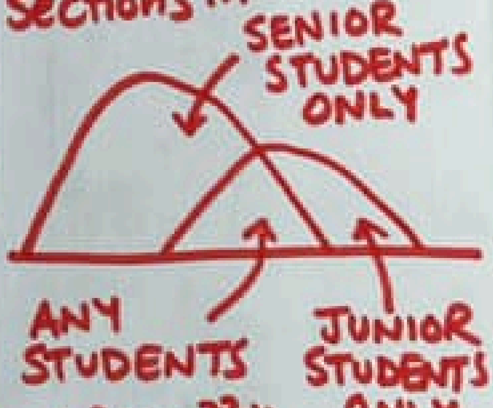
b) Find a simplified expression for $f(x-1)$

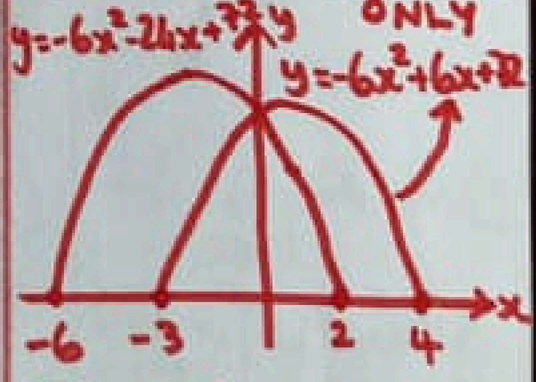
c) Hence simplify $\dfrac{f(x+1)-f(x-1)}{6}$

⑧ Show that $\dfrac{\sin 2A}{\tan A}=2\cos^2 A$

⑨ Find the coordinates of the stationary points of $y=x^4-8x^2+3$ and determine their nature.

⑩ A school splits its lunch hall into three sections...





The sections are represented by

$y=-6x^2-24x+72$

and

$y=-6x^2+6x+72$

Find the area of the three sections.

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①

$\sqrt{3}\sin x° + \cos x°$

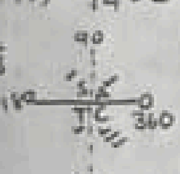
$= k\sin x\cos a - k\cos x\sin a$

$k\cos a=\sqrt{3}$

$k\sin a=-1$

$k=\sqrt{(\sqrt{3})^2+(1)^2}=\sqrt{4}=2$

$\tan a=-\frac{1}{\sqrt{3}}$



$a=360-30=330$

So

$\sqrt{3}\sin x° + \cos x°$

$=\underline{2\sin(x-330)°}$

b) maximum value $=\underline{2}$

②

$\underline{a}.\underline{b}=3(-1)+(-1)(-5)+4(9)$

$=-3+5+36$

$=38$

$|\underline{a}|=\sqrt{3^2+(-1)^2+4^2}=\sqrt{26}$

$|\underline{b}|=\sqrt{(-1)^2+(-5)^2+9^2}=\sqrt{107}$

$\underline{a}.\underline{b}=|\underline{a}||\underline{b}|\cos\theta$

$\Rightarrow \cos\theta=\dfrac{38}{\sqrt{26}\sqrt{107}}$

so $\theta=43.9°$

③ sub $y=x+4$ into circle

$x^2+(x+4)^2+8x+2(x+4)-96=0$

$x^2+x^2+8x+16+8x+2x+8-96=0$

$2x^2+18x-72=0$

$x^2+9x-36=0$

$(x+12)(x-3)=0$

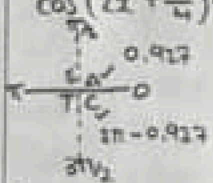
$x=-12$ $\quad$ $x=3$

$y=(-12)+4=-8$ $\quad$ $y=3+4=7$

intersect at $(-12,-8)$ and $(3,7)$

④

$\cos\left(2x+\frac{\pi}{4}\right)=\frac{3}{5}$



$2x+\frac{\pi}{4}=0.927,$
$5.356,$
$7.210,$
11.639

$2x=0.142$
4.571
6.425
10.854

$x=0.07, 2.29$
$3.21, 5.43$

⑤ real roots if $b^2-4ac\geq 0$

$a=1, b=(p+1), c=9$

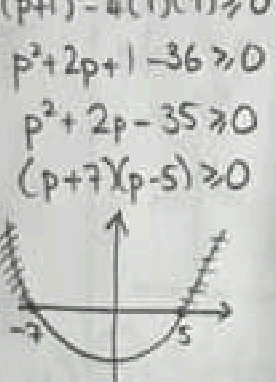
$b^2-4ac\geq 0$

$(p+1)^2-4(1)(9)\geq 0$

$p^2+2p+1-36\geq 0$

$p^2+2p-35\geq 0$

$(p+7)(p-5)\geq 0$



$\underline{p\geq 5}$ or $\underline{p\leq -7}$

⑥

a) $t=0$

$\Rightarrow T_0=4000e^0$

$=4000$

There were 4000 in the year 2000.

b) $3200=4000e^{-10k}$

$0.8=e^{-10k}$

$\ln 0.8=-10k$

$k=\dfrac{\ln 0.8}{-10}$

$k=0.0223$

$T_{30}=4000e^{-0.0223\times 30}$

$T_{30}=\underline{2049\text{ tigers}}$

⑦

a) $3(x+1)^2+6(x+1)+2$

$=3(x^2+2x+1)+6x+6+2$

$=3x^2+6x+3+6x+8$

$=3x^2+12x+11$

as required.

b) $3(x-1)^2+6(x-1)+2$

$=3(x^2-2x+1)+6x-6+2$

$=3x^2-6x+3+6x-4$

$=3x^2-1.$

c) $\dfrac{3x^2+12x+11-(3x^2-1)}{6}$

$=\dfrac{12x+12}{6}$

$=\underline{2x+2}$

⑧

$2\sin A\cos A \div \tan A$

$2\sin A\cos A \div \dfrac{\sin A}{\cos A}$

$2\sin A\cos A \times \dfrac{\cos A}{\sin A}$

$=\underline{2\cos^2 A}$

⑨ SPs occur when $\frac{dy}{dx}=0$

$\frac{dy}{dx}=4x^3-16x$

$4x^3-16x=0$

$4x(x^2-4)=0$

$x=0$ $\quad$ $x=\pm 2$

x	$\rightarrow -2 \rightarrow 0 \rightarrow 2 \rightarrow$
$\frac{dy}{dx}$	$-\ 0\ +\ 0\ -\ 0\ +$
	↘ → ↗ → ↘ → ↗

$x=-2 \Rightarrow y=3$

$x=0 \Rightarrow y=-13$

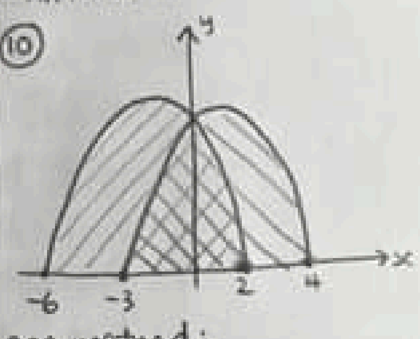
$x=2 \Rightarrow y=-13$

minimum TP@ $(-2,3)$

maximum TP@ $(0,-13)$

minimum TP@ $(2,-13)$

⑩



one method:

Area from -6 to 2 $=\displaystyle\int_{-6}^{2}-6x^2-24x+72\,dx$

$=\left[-2x^3-12x^2+72x\right]_{-6}^{2}$

$=512$

Area of ▨ in $x>0$ side $=\displaystyle\int_{0}^{2}-6x^2-24x+72\,dx$

$=\left[-2x^3-12x^2+72x\right]_{0}^{2}$

$=80$

Area from -3 to 4 $=\displaystyle\int_{-3}^{4}-6x^2+6x+72\,dx$

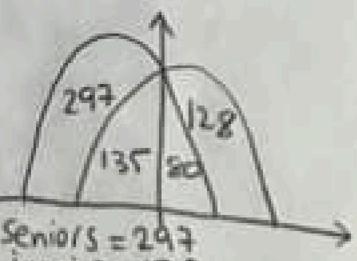
$=\left[-2x^3+3x^2+72x\right]_{-3}^{4}$

$=343$

Area of ▨ in $x<0$ side $=\displaystyle\int_{-3}^{0}-6x^2+6x+72\,dx$

$=\left[-2x^3+3x^2+72x\right]_{-3}^{0}$

$=135$



Seniors = 297

Juniors = 128

any = 215